Determination of GPS Positioning Errors due to Multi-Path in Civil Aviation

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Abstract—**Fully automated landing systems based on GPS still suffer from errors, mainly due to multipath propagation. This paper presents a simulation tool to quantify positioning errors by combining a ray-tracing based three-dimensional multipath propagation model with a standard GPS receiver model. The propagation model accounts for the reflecting environment geometry, transmitting and receiving antenna patterns, reflections and depolarization. For each ray the time delay, carrier phase delay and multipath-to-direct amplitude ratio is computed and fed into the receiver model to determine the pseudorange errors. The performance of the method is illustrated with numerical examples.**

I. Introduction

The Global Positioning System (GPS) is widely seen as the successor to many navigational applications used in civil aviation today [1]. For example, GPS has proven to be effective for en-route navigation. However, fully automated landing systems based on GPS still suffer from errors, mainly due to multipath propagation of the signal [2]. Multipath is caused by reflection or scattering from objects in the neighborhood of the GPS receiver, including the ground. Also the shape of the airplane itself can give rise to multipath, for example by reflections from wings and tail. Although several techniques have been proposed to reduce the effects of multipath [3]-[5], it remains a limiting factor for GPS. In this paper a software tool is developed to investigate positioning errors introduced by multipath by means of numerical simulations for a static configuration [6]. Therefore it is necessary to model on the one hand the multi-path electromagnetic propagation in a given three-dimensional (3D) reflecting environment and on the other hand the reception and signal processing in the GPS receiver. An image based ray-tracing algorithm has been implemented to compute the possible pathways between transmitters and receiver. This algorithm accounts for arbitrary polarizations of the waves and for reflections at surfaces with given electromagnetic material parameters. Changes in the RHCP polarization of the transmitted wave thus are accurately computed. Also the antenna pattern of the antennas are taken into account. As a result the time delay, carrier phase delay and multipath-to-direct amplitude ratio are computed for each multipath ray. These parameters then are used to quantify the multipath error introduced in the tracking loops. A coherent receiver with a classical discriminator function is considered. In [7] the effects of multipath on GALILEO performance in an urban environment were examined with a ray-launching technique for a non coherent delay lock loop.

A formulation of the problem is given in Section II. The signal processing part in the GPS receiver is described in Section III and the multipath propagation model is discussed in Section IV. Numerical results are given in Section V.

II. Formulation

Consider a transmitting satellite antenna and an aircraft-mounted receiving antenna with coordinates $\mathbf{r}_t = (x_t, y_t, z_t)$ and $\mathbf{r}_r = (x_r, y_r, z_r)$, respectively, with respect to an earth based reference system $oxyz$. The z-axis is chosen sufficiently close to the receiver, such that the xy-plane is a good approximation to the (curved) mean sea level in the environment of the receiver. The height of the receiver above the earth surface is denoted with H and the elevation of the satellite with respect to the xy-plane with ψ. The line-of-sight (LOS) electric far field of the transmitter in an observation point $\mathbf{r}$ is given by

$$\mathbf{E}_{los}(\mathbf{r},\omega) = \mathbf{F}_t(\mathbf{u},\omega)\frac{e^{-jkR}}{R}, \tag{1}$$

where $\omega = 2\pi f$ is the angular frequency, $\mathbf{F}_t$ is the complex radiation vector of the transmitting antenna, $k = \omega/c$ is the free space wavenumber, with c the velocity of light, $R = |\mathbf{r} - \mathbf{r}_t|$ is the distance from the transmitter to the observation point and $\mathbf{u} = (\mathbf{r} - \mathbf{r}_t)/R$ is the unity vector along the observation direction. The total field $\mathbf{E}(\mathbf{r}_r)$ at the receiver consists of a LOS contribution $\mathbf{E}_{los}(\mathbf{r}_r)$ and K multipath contributions $\mathbf{E}_{m,k}(\mathbf{r}_r)$, $k = 1...K$, e.g. from reflections by buildings, the surface of the earth,... In this paper a software code has been developed to quantify the multipath error on the position x_r, y_r, z_r by combining a GPS receiver model and a multipath propagation model.

In the transmitter, a satellite specific Pseudo Random Noise (PRN) code $p(t) = \pm 1$ with chip T is modulated with Binary Phase Shift Keying (BPSK) on a RF carrier with angular frequency ω_c, yielding a signal

$$s_t(t) = A_t\, p(t) cos(\omega_c t + \theta_t), \tag{2}$$

which is transmitted as a Right Hand Circularly Polarized (RHCP) wave. The analysis in this paper is limited to the $L1$ = 1575.42 MHz carrier frequency and to the C/A code,

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which is a 1023 bit sequence with chip T = 977.5 ns and clock rate $1/T$ = 1023 MHz. One chip thus corresponds to a propagation distance of cT = 293 m. The 50-bits/s navigation message is not included in (2), since it is not useful further in the analysis. The RF LOS signal at the receiver is given by

$$s_{los}(t) = A\,p(t - \Delta t)cos\omega_c t, \tag{3}$$

where A is the LOS signal amplitude, $\Delta t = R/c$ is the time delay between the received and transmitted code and where for convenience the carrier phase reference is chosen equal to zero. To determine the receiver position (x_r, y_r, z_r) measurements of LOS pseudoranges $c\Delta t_i$ for at least 3 satellites in positions $(x_{t,i}, y_{t,i}, z_{t,i})$ are needed [8]. A multipath ray travels an additional distance ΔR_m with respect to the LOS ray and experiences reflections and depolarization, such that a multipath signal at the receiver is given by

$$s_m(t) = \alpha A\,p(t - \Delta t - \Delta t_m)cos(\omega_c t - \theta_m), \tag{4}$$

where α is the multipath-to-direct amplitude ratio, $\Delta t_m = \Delta R_m/c$ is the additional time delay with respect to the LOS signal and $\theta_m = \omega\Delta t_m - \theta_{ref}$ is the carrier phase delay, with θ_r the phase change due to reflections. The total signal $s(t)$ is the sum of (3) and K multipath signals (4)

$$s(t) = s_{los}(t) + \sum_{k=1}^{K} s_{m,k}(t). \tag{5}$$

In this paper the multipath parameters $\Delta t_{m,k}$, α_k and $\theta_{m,k}$ are computed with a ray tracing technique for a given configuration of the environment, transmitter and receiver (see section IV), while an estimation of the resulting error in the pseudorange is performed with a standard receiver model (see section III).

For a receiver with a typical correlator spacing $2\tau_d = T$ it can be shown that multipath signals that arrive with a delay $\Delta t_m > 1.5T$ are rejected. For the C/A code this delay is 1466 ns and corresponds to a pathlength of 440m. For the configuration of a satellite with elevation ψ and a receiver at a height H above a flat ground, reflections cannot be neglected if $Hsin\psi < 220$ m. A receiver below H_{min} = 220 m thus is disturbed by multipath independent of the satellite elevation. The value of H_{min} can be reduced by the use of a narrow correlator [9].

III. Receiver

In this paragraph the operations performed by the tracking loops, more particularly the Phase Lock Loop (PLL) and Delay Lock Loop (DLL), in the signal processing part of a standard GPS receiver are discussed [4][10]. The input s_{if}, which is obtained by down conversion of (5) in the receiver front-end, is given by

$$\begin{aligned} s_{if}(t) &= p(t - \Delta t)cos\omega_{if} t, \\ &\quad +\alpha p(t - \Delta t - \Delta t_m)cos(\omega_{if} t - \theta_m), \end{aligned} \tag{6}$$

where ω_{if} is the intermediate frequency and where—for convenience—only one multipath signal is considered (K = 1), the amplitude A is omitted, since only the multipath-to-direct ratio is important in the following, and A/D sampling is not accounted for (an analogue description instead of a digital one is being used). In absence of multipath, the tracking loops generate local replica of the carrier and code, that are in phase with the incoming LOS signal. Multipath causes the replica to be out of phase with the LOS signal.

Let $R(\tau) = \overline{p(t)p(t+\tau)}$ be the autocorrelation function of the code $p(t)$, where over bar stands for time averaging. In this paper, $R(\tau)$ is given by

$$\begin{aligned} R(\tau) &= 1 - \tfrac{|\tau|}{T}, & |\tau| \le T, \\ &= 0, & |\tau| > T. \end{aligned} \tag{7}$$

Consider prompt, early and late versions of the locally generated code

$$p_p(t + \tau) = p(t - \Delta t + \tau), \tag{8}$$

$$p_e(t + \tau) = p(t - \Delta t + \tau - \tau_d), \tag{9}$$

$$p_l(t + \tau) = p(t - \Delta t + \tau + \tau_d), \tag{10}$$

where τ is the DLL aligning error with respect to the LOS signal and where $2\tau_d$ is the correlator spacing.

The local oscillator in the PLL generates—when it is locked—a signal

$$s_{pll}(t) = cos(\omega_{if} t + \theta_c), \tag{11}$$

which has the same frequency and phase as the incoming carrier, hence θ_c is the phase of the composite signal (6). Locking typically is achieved with a Costas loop. Multiplication of (11) with (6) and low-pass filtering yields the in-phase baseband component

$$\begin{aligned} I_{bb}(t) &= p(t - \Delta t)cos\theta_c \\ &\quad +\alpha p(t - \Delta t - \Delta t_m)cos(\theta_m - \theta_c). \end{aligned} \tag{12}$$

Similarly, the quadrature component $Q_{bb}(t)$ is obtained from the 90° phase shifted local oscillator. In the Costas loop, the correlations $I_p(\tau)$ and $Q_p(\tau)$ of $I_{bb}(t)$ and $Q_{bb}(t)$, respectively, with the prompt code $p_p(t)$ are used to control the local oscillator. Locking of (11) is achieved when $Q_p(\tau) = 0$, hence

$$\theta_c = \arctan\left(\frac{\alpha R(\tau + \Delta t_m)\sin\theta_m}{R(\tau) + \alpha R(\tau + \Delta t_m)\cos\theta_m}\right). \tag{13}$$

Note that $\theta_c = 0$ in absence of multipath .

In absence of multipath, the code generator in the DLL generates—when it is locked—a prompt code $p_p(t)$ that is aligned with the incoming code $p(t - \Delta t)$, hence the DLL phase error $\tau = 0$ in (8). Therefore, the code generator is controlled by a discriminator function, that takes a zero value for $\tau = 0$. In this paper, a classical coherent discriminator function $D(\tau) = I_e(\tau) - I_l(\tau)$ is adopted.

In the presence of multipath, the normalized discriminator function is given by

$$\begin{aligned} D(\tau) &= [R(\tau + \tau_d) - R(\tau - \tau_d)]\cos\theta_c \\ &\quad +\alpha\,[R(\tau + \tau_d + \Delta t_m) - R(\tau - \tau_d + \Delta t_m)] \\ &\quad \cos(\theta_m - \theta_c), \end{aligned} \tag{14}$$

which generally takes a zero value for an aligning error $\tau \neq 0$. This zero is determined with the algorithm of [11]. The resulting error on the pseudorange then is given by $\Delta R = -c\tau$.

IV. MULTIPATH PROPAGATION

A. Raytracing

The 3D multipath reflecting environment (ground, buildings,...) is modeled by means of polygons. Each polygon is an interface between the air and a medium with given electromagnetic material parameters ε, μ. Since propagation distances and reflecting obstacles are large with respect to the wavelength—for L1, λ = 19 cm—a geometrical optics approach is applied. A so-called "image-based" ray-tracing technique [12] is used to compute all possible ray paths between the transmitter and receiver. With this method the maximum number of reflections P is selected in advance and all mirror points that yield rays, which reach the receiver after maximum P reflections are computed. Next the coordinates of the extremities of the consecutive straight sections that make up each multipath ray are calculated and the polarization dependent reflection coefficients are computed for the relevant polygons. For each ray k, the time delay $\Delta t_{m,k}$ then is derived from the additional distance $\Delta R_{m,k}$ with respect to the LOS path length, the carrier phase delay is computed from $\Delta R_{m,k}$ and from the total phase change $\theta_{ref,k}$ due to reflections and also the multipath-to-direct ratio α_k is obtained from the reflections. Note that the attenuation factor $1/R$ in (1) can be neglected, since $\Delta t_{m,k}$ < 1466 ns $\ll \Delta t$. Propagation between two points separated by a distance R thus is modeled by multiplying the complex amplitude of $\mathbf{E}$ with the phase factor e^{-jkR} only.

B. Transmitting RHCP waves

The RHCP polarization of the transmitted wave can be made fairly uniform over the main lobe of the emitted beam (pointing to the earth's center), when choosing a vector $\mathbf{w}$ that is parallel to the surface of the earth below the satellite and when letting the radiation vector $\mathbf{F}_t$ of the transmitting antenna be given by

$$\mathbf{F}_t(\mathbf{u}) = F_t\mathbf{v}_1 - jF_t\mathbf{v}_2, \tag{15}$$

where F_t is a constant and

$$\mathbf{v}_1 = \frac{\mathbf{u}\times\mathbf{w}}{|\mathbf{u}\times\mathbf{w}|}, \tag{16}$$

$$\mathbf{v}_2 = \mathbf{u}\times\mathbf{v}_1. \tag{17}$$

For every ray that reaches the receiver, the direction $\mathbf{u}$ is determined from the coordinates of the transmitter and those of the receiver (LOS-ray) or of the first reflection point (multipath rays).

C. Reflections

Whereas the polarization of the LOS wave remains unchanged, multipath waves generally are depolarized, since the $TE-$ and $TM-$polarizations that compose the RHCP wave are reflected differently. Consider a multipath ray along a direction $\mathbf{u}_i$, that is incident on a polygon with normal $\mathbf{n}$ and relative permittivity ε_r. The wave is decomposed in a TE-polarization—this is the component along the unity vector $\mathbf{v}_\perp$, which is perpendicular to the plane of incidence—and a TM-polarization—this is the component along the unity vector $\mathbf{v}_\parallel = \mathbf{v}_\perp \times \mathbf{u}_i$, which is in the plane of incidence,

$$\mathbf{E}_{i,\perp} = (\mathbf{E}_i\cdot\mathbf{v}_\perp)\mathbf{v}_\perp, \tag{18}$$

$$\mathbf{E}_{i,\parallel} = \mathbf{E}_i - \mathbf{E}_{i,\perp}. \tag{19}$$

The reflected wave propagates along $\mathbf{u}_r$ and is composed of

$$\mathbf{E}_{r,\perp} = K_\perp\mathbf{E}_{i,\perp}, \tag{20}$$

$$\mathbf{E}_{r,\parallel} = K_\parallel(\mathbf{E}_{i,\parallel}\cdot\mathbf{v}_\parallel)\mathbf{v}_\perp\times\mathbf{u}_r, \tag{21}$$

where $K_\perp$ and $K_\parallel$ are the Fresnel reflection coefficients.

D. Determination of the RHCP-component at the receiver

Due to depolarization multipath rays generally are composed of an RHCP and a LHCP component. Knowledge of these components allows to evaluate the reception for a given GPS receiving antenna. A coordinate system $o'x'y'z'$ is associated with the receiving antenna. Let

$$\mathbf{u}_{\phi'} = \frac{\mathbf{u}\times\mathbf{u}_{z'}}{|\mathbf{u}\times\mathbf{u}_{z'}|}, \tag{22}$$

$$\mathbf{u}_{\theta'} = \frac{\mathbf{u}\times\mathbf{u}_{\phi'}}{|\mathbf{u}\times\mathbf{u}_{\phi'}|}, \tag{23}$$

where $\mathbf{u}$ is the propagation direction of an incoming wave. Projection of the field on $\mathbf{u}_{\phi'}$ and $\mathbf{u}_{\theta'}$ yields two orthogonal, linearly polarized components $E_{\phi'} = \mathbf{E}\cdot\mathbf{u}_{\phi'}$ and $E_{\theta'} = \mathbf{E}\cdot\mathbf{u}_{\theta'}$. The circularly polarized components are given by

$$E_{RHCP}(\theta',\phi') = \frac{E_{\phi'} + jE_{\theta'}}{2}, \tag{24}$$

$$E_{LHCP}(\theta',\phi') = \frac{E_{\phi'} - jE_{\theta'}}{2}. \tag{25}$$

V. NUMERICAL RESULTS

The influence of one multipath ray on the discriminator function (14), plotted in Fig. 1 as a function of the normalized time τ/T, is illustrated for two different time delays Δt_m. The correlator spacing is 1 chip, or $\tau_d = 0.5T$. The plain line is for a ray with a small delay $\Delta t_m = 0.05T$ with $\theta_m = 0$ and $\alpha = 1$ and the dotted line is for a ray with a larger delay $\Delta t_m = 0.5T$ with $\theta_m = \pi$ and $\alpha = 0.5$. For this latter case, the zero of the discriminator function clearly shows a shift to the right.

Next the error on the pseudorange is examined as a function of the receiver height H above a flat dry ground. Fig. 2 shows

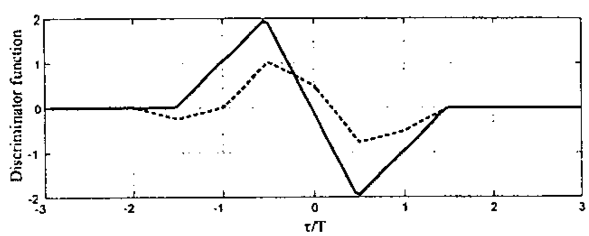

Fig. 1. Normalized discriminator function as a function of normalized time for: (—) ray with $\Delta t_m = 0.05T$, $\theta_m = 0$, $\alpha = 1$ and (- - -) $\Delta t_m = 0.5T$, $\theta_m = \pi$, $\alpha = 0.5$.

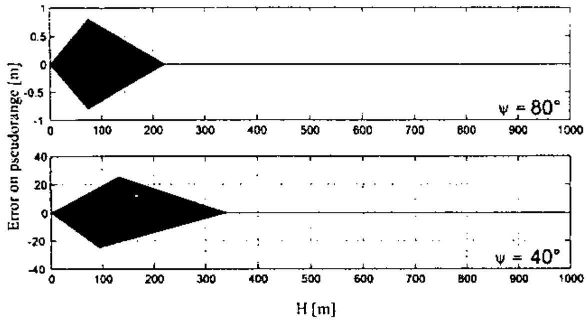

Fig. 2. Error on pseudorange as a function of receiver height for satellites with elevation 80 ° and 40 °, $\tau_d = 0.5T$

results for satellite elevations of $\psi = 80°$ and $40°$. The satellite with the lowest elevation yields errors up to 25 m below a receiver height of 330m.

In Fig. 3 this experiment is repeated for a receiver with a correlator spacing of 10 times smaller, $\tau_d = 0.05T$. The maximum error is also reduced by a factor 10.

Fig. 4 shows the configuration of three satellite transmitters and a receiver at height H in the neighborhood of a building and a hill. The LOS and multipath rays are indicated as well. The resulting errors on the pseudoranges as a function of the receiver height are shown for two of the satellites in Fig. 5. These errors then can be used in the algorithm [8] to determine the xyz-positioning errors.

VI. Conclusion

A method was presented to quantify the pseudorange error introduced by multipath, by combining a ray-tracing based multipath propagation model with a standard GPS receiver model. The software can be used to perform accurate simulations for realistic configurations of the environment and the transmitting and receiving antennas and to investigate the effect of varying receiver parameters on the multipath error.

References

[1] B.W. Parkinson, J.J. Spilker, P. Axelrad and P. Enge, Eds., *Global Positioning System: Theory and Applications*, vol. II, Washington DC: American Institute of Aeronautics and Astronautics, 1996.

[2] S. Braasch,"Multipath Effects," in *Global Positioning System: Theory and Applications*, vol. I, B.W. Parkinson, J.J. Spilker, P. Axelrad and P. Enge, Eds. Washington DC: American Institute of Aeronautics and Astronautics, 1996, pp. 547–568.

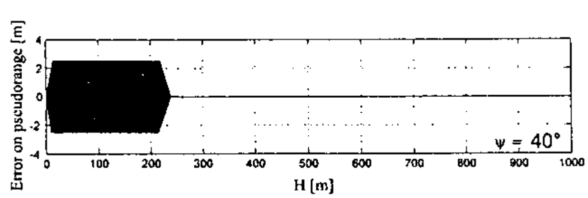

Fig. 3. Error on pseudorange as a function of receiver height for satellite with elevation 40 ° and narrow correlator $\tau_d = 0.05T$.

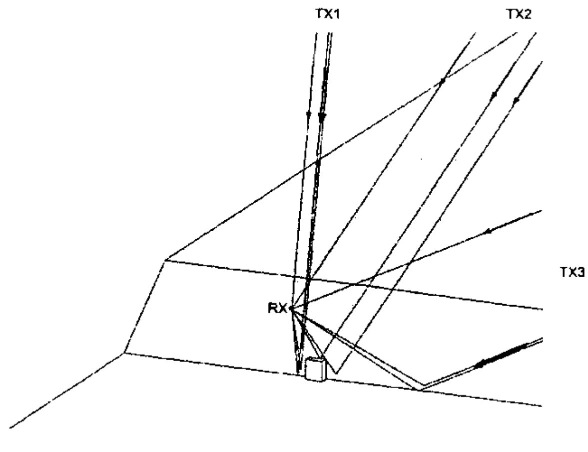

Fig. 4. Rays in configuration with 3 satellites and a receiver near a hill.

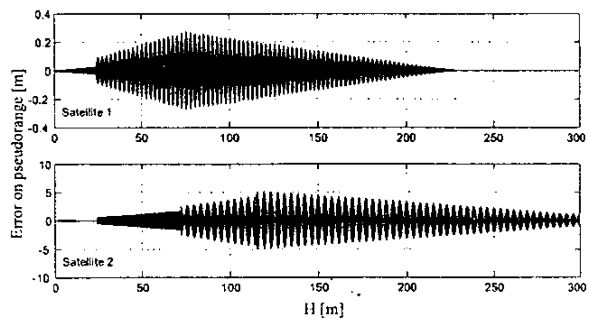

Fig. 5. Error on pseudorange as a function of receiver height for satellites 1 and 2 of the configuration of Fig. 4.

[3] J.K. Ray, M.E. Cannon and P. Fenton, "GPS Code and Carrier Multipath Mitigation Using a Multiantenna System," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 37, pp. 183–195, Jan. 2001.

[4] S. Braasch and A.J. Van Dierendonck, "GPS Receiver Architectures and Measurements," *Proceedings of the IEEE*, vol. 87, nr. 1, 1999, pp. 48–64.

[5] P. Fenton, B. Falkenberg, T. Ford, K. Ng and A.J. Van Dierendocnk, "NovAtel's GPS receiver - the high performance OEM sensor of the future," *Proc. of ION GPS-91*, Albuquerque, Sept. 9-13 1991, pp. 49–58.

[6] L. Roelens,"Modellering van de invloed van omgevingsfactoren op een avionica GPS systeem met behulp van raytracing," Masters Thesis, Ghent University, 2004.

[7] M. Malicorne, M. Bousquet, V. Calmettes and C. Macabiau, "Influence of Multipath on GNSS Receiver Performances in Urban Environment," *8th Saint Petersburg Conference on Integrated Navigation Systems*, 2001.

[8] N. Crocetto, M.G. D'Urso and M. Gatti, "Processing of pseudorange measurements: an exact and an iterative algorithm for the GPS single point positioning," *Proc. Workshop Int. Cooperation and Technology Transfer*, ISPRS Com. VI, Working Group 3, Febr. 1998, pp. 134–141.

[9] A. Van Dierendonck, P. Fenton and T. Ford, "Theory and Perfomance of Narrow Correlator Spacing in a GPS Receiver," *Navigation*, vol. 39, nr. 3, 1992, pp. 256–283.

[10] C. Macabiau, B. Roturier and A. Benhallam, "Performance of GPS Receivers with more than one Multipath," *Proc. ION GPS-99*, Nashville, 1999.

[11] R.P. Brent, "An algorithm with guaranteed convergence for finding a zero of a function," *The Computer Journal* vol. 14, March 1971, pp. 422–425.

[12] B. De Backer, H. Borjeson, D. De Zutter and F. Olyslager, "Propagation mechanisms for UHF wave transmission through walls and windows," IEEE Trans. Vehicular Technology, vol. 52, Sept. 2003, pp. 1297–1307.

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